Strategies for turbulence modelling and simulations

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This is an attempt to clarify and size up the many levels possible for the numerical prediction of a turbulent flow, the target being a complete airplane, turbine, or car. Not all the author's opinions will be accepted, but his hope is to stimulate reflection, discussion, and planning. These levels still range from a solution of the steady Reynolds-Averaged Navier-Stokes (RANS) equations to a Direct Numerical Simulation, with Large-Eddy Simulation in-between. However recent years have added intermediate strategies, dubbed "VLES", "URANS" and "DES". They are in experimental use and, although more expensive, threaten complex RANS models especially for bluff-body and similar flows. Turbulence predictions in aerodynamics face two principal challenges: (I) growth and separation of the boundary layer, and (II) momentum transfer after separation. (I) is simpler, but makes very high accuracy demands, and appears to give models of higher complexity little advantage. (II) is now the arena for complex RANS models and the newer strategies, by which time-dependent three-dimensional simulations over two-dimensional geometries are the norm. In some strategies, grid refinement is aimed at numerical accuracy; in others it is aimed at richer turbulence physics. In some approaches the empirical constants play a strong role even when the grid is very fine; in others, their role vanishes. For several decades, practical methods will necessarily be RANS, possibly unsteady, or RANS/LES hybrids, pure LES being unaffordable. Their empirical content will remain substantial, and the law of the wall will be particularly resistant. Estimates are offered of the grid resolution needed for the application of each strategy to full-blown aerodynamic calculations, feeding into rough estimates of its feasibility date, based on computing-power growth.

1. INTRODUCTION

The turbulence problem is of course far from solved, whether in terms of mathematical and intuitive understanding, or in terms of obtaining engineering accuracy for machines that depend on viscous fluid dynamics. Technological fields of global importance such as the airliner and automobile industries revolve around such devices. This economic stake motivates relentless, imaginative, and expensive efforts at turbulence prediction by any plausible approach. This should not defeat common sense or let us hide from cost estimates [49], and we must have visibility of when a method may progress from research to engineering. Chapman made such predictions in 1979 [9], which still carry weight although his view of turbulence prediction in the 1990's is now recognized as optimistic.

This paper focuses on the *numerical* prediction of *turbulent* flow regions. The equally difficult problem of transition prediction is mentioned only in passing. Physical testing methods in the transportation industry are beset by their own severe transition- and turbulence-related difficulties. Tests with scale models usually imply both lower Reynolds numbers and/or higher freestream turbulence levels (in addition to blockage, surface-quality, bracket and mounting issues, and aero-elastic differences). The resulting scale effects can be misleading, and unforeseen reversals of the normal trend (by which higher Reynolds numbers bring better performance) are very damaging. This is especially true as competing companies seek optimal aerodynamic designs. Such designs have narrow margins, and magnify the sensitivity to viscous effects. In view of the limits on testing accuracy, industry demands accuracy from Computational Fluid Dynamics (CFD), but not perfection.

Also note that, whether in the airframe, turbine engine, or automotive industry, turbulence is not the only obstacle in CFD. Major numerical challenges remain between the state of the art and the routine calculation of flows over even rather simple 3D geometries. These challenges relate not only to computing cost, but also to solution quality, particularly in terms of gridding. Presenting turbulence as the only "pacing item" in CFD could benefit research funding, but it is not accurate. On the other hand, it sometimes appears that more capable people are engaged in grid generation, solvers, and pre- and post-processors, than in turbulence. Our effort may be unbalanced, although more duplication occurs on the programming side (it is easier to show progress in programming, not to mention code exercising, than in modelling). Sharing large codes is more difficult than sharing turbulence models, for which the equations (normally!) fit on one page. With a few exceptions, models have been freely published.

The numerical strengths of CFD increase by the year thanks to the progress of computers, whereas turbulence modelling *can* stagnate. If that takes place, modelling will become the principal pacing item in some types of industrial CFD in less than a decade, at least in the Reynolds Averaged Navier-Stokes (RANS) frame. It is then very sensible to examine approaches that trade high-level turbulence theories ("intelligence") for computing effort (unfairly described as "brute force"). A primary purpose of this paper is to provide a viewpoint on these relatively new, evolving and misunderstood methods, besides predicting that they will proliferate and make a major contribution. The most stimulating issue may be the share between empiricism and numerical power in the successful methods (§2). The concrete cost issues are addressed through a table attached to §3.

2. PHYSICAL ASPECTS

2.1. RANS models

The field of classical RANS turbulence modelling is active. At a recent biennial international symposium, about twenty-five papers presented new models or new versions of models [15]. These were offered for outside use, with varying degrees of completeness in the description. No student of turbulence has the time to give each of these serious consideration. The full range of RANS methods is receiving work; this unfortunately testifies that no class of models has emerged as clearly superior, or clearly hopeless. Activity is not even restricted to differential methods; isolated groups are refining integral boundary-layer solvers, to allow more three-dimensionality and more separation. The same seems to apply to algebraic models. Eddy-viscosity transport models, being the simplest models that can be applied with a general grid structure, are now used extensively. The step back from two equations to a single equation has not crippled the approach [20,45,48,56], while tangibly reducing the true cost of solutions (partly by allowing a coarser grid spacing at the wall). Conversely, models with up to four equations are in contention [14,39]. Perot's is especially intriguing.

The abstract referred to "Challenges I and II"; the following "Challenge Zero" could be added. Complete configurations often have laminar regions in their boundary layers; it is very helpful if a turbulence model can be "dormant" in such regions, meaning that its transport equations accept solutions with vanishing Reynolds stresses. Similarly, regions of irrotational and non-turbulent fluid, which are large in external aerodynamics, do not physically influence the turbulent regions such as boundary layers (weak freestream activity does have much influence on natural transition, but we leave transition prediction to a separate method). Again it is very helpful if the model accepts zero values in such regions, or small values without influence on the turbulent layers. At the same time, the model should allow the contamination of a laminar shear layer by contact with a turbulent shear layer (transition triggered by moderate freestream turbulence is more subtle, and is within reach of only a few models). This all depends on the behaviour of the model at the turbulent/non-turbulent interface. In some models the stress level in the turbulent layer depends demonstrably on either the freestream values of the turbulence variables or, even worse, on the grid spacing at the interface. Few people have devoted attention to this question [30,8,48], and model descriptions sometimes make no mention of recommended freestream values (and also fail to demonstrate insensitivity). However, it happens that the k-ϵ, SST and S-A models, which all three are insensitive to freestream values, can fairly be described as "popular". Their results are reproducible from code to code and grid to grid. In the perennial question of the choice of a second variable in two-equation models, freestream sensitivity should be given a high priority. It is much more important than the value of some high derivative at the wall.

The failure of most models to predict relaminarisation also causes frustration. While it is not reasonable to expect a model to predict transition in quiet environments, expecting relaminarisation is rather justified.

In terms of Challenge I, the different classes of models are surprisingly even, in the sense that the best models in each class perform quite comparably. Within that challenge, we can include the prediction of skin friction and boundary-layer thicknesses (which dictate the parasite drag in the absence of separation), along with separation (which creates pressure drag). Integral methods and algebraic boundary-layer models have been so well optimised that surpassing them with any Navier-Stokes model is difficult. Reasons include the grid needed, the intrusion of artificial dissipation, and the constraints placed on the turbulence model such as locality, performance in free shear flows, and simplicity. It is geometric complexity and the drive towards massive separation, not lack of accuracy, that are making integral and algebraic methods obsolete.

2.2. Simple RANS models

We are referring here primarily to eddy-viscosity models. Improvements will be made to the simpler transport models, typically by adding new empirical terms aimed at compressibility, streamline curvature or better anisotropy of the Reynolds-stress tensor (non-Boussinesq constitutive relations) which can, for instance, create secondary flows of the second kind in a square pipe [53]. An example is given in fig. 1. The S-A model (see Appendix) was used with the following constitutive relation, as a first attempt to improve on the eddy-viscosity approximation. Let $\overline{\tau}_{ij}$ be the Reynolds stress given by the linear model. The nonlinear stress is then

$$\tau_{ij} = \overline{\tau}_{ij} - c_{nl1}\left[O_{ik}\overline{\tau}_{jk} + O_{jk}\overline{\tau}_{ik}\right]$$

where

$$O_{ik} \equiv \frac{\partial_k U_i - \partial_i U_k}{\sqrt{\partial_n U_m \partial_n U_m}}$$

is the normalized rotation tensor. The constant $c_{nl1} = 0.3$ was calibrated in the outer region of a simple boundary layer, by requiring a fair level of anisotropy $\overline{u'^2} > \overline{w'^2} > \overline{v'^2}$ (the streamwise, spanwise and wall-normal Reynolds stresses, respectively). The result in the square duct is quite positive: flow is induced towards the corners, and the skin friction is much closer to experiment [19]. However, other flows such as 3D wall jets have led to negative results (A. N. Secundov, personal communication, 1999). The c_{nl1} term must also be considered as very preliminary, in the sense that it uses only one of the many quadratic combinations of strain and vorticity. Also note that it is fully empirical, instead of being derived from a more complex model; we simply selected the most intuitively attractive combination. A systematic optimisation has not been performed. Nevertheless, it is of interest to show that even a one-equation model can be made to predict this secondary flow. Similarly, realisable versions of simple models can be created (note that the common one-equation models are far from giving realisable Reynolds-stress tensors), but we have found the effect too weak to justify a widespread modification of codes and testing campaign.

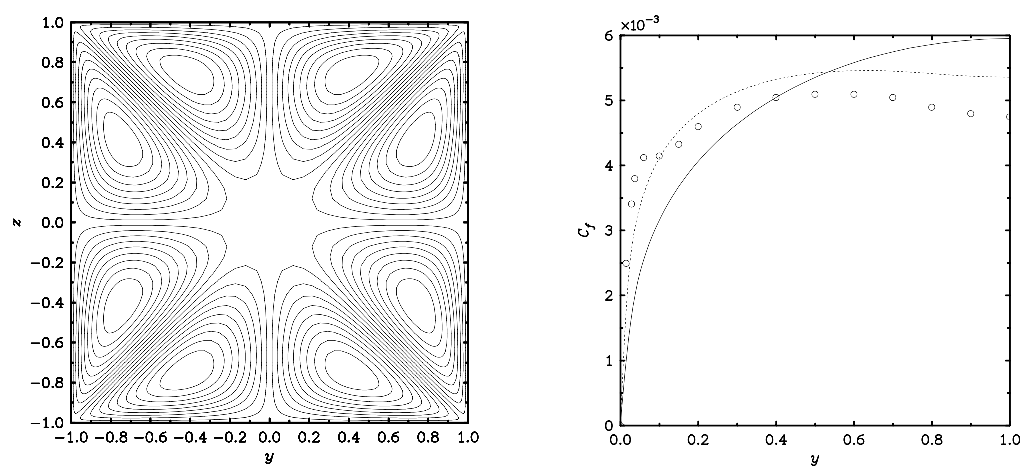

Figure 1. Square channel, with nonlinear constitutive relation. Left, streamlines in cross plane. Right, skin-friction coefficient. —, linear model; - - -, nonlinear; ∘, Exp.

The activity of upgrading a widespread model is and should be very cautious, as it is highly desirable for new versions to preserve all or almost all of the past successes of a model. In rough terms, we hope for gradual progress on Challenge II but are unwilling to give up any of the accuracy and experience base in Challenge I. For this reason, this author is very intent on limiting the number of versions of the S-A model, believing that it best serves the needs of the community. In addition, the rate at which new models or even versions are added to government and industry 3D codes is unfortunately very slow. Codes become so large they are difficult to manage and to test, and frequent changes in computer architecture divert the attention of the code custodians. This could cause a model to become entrenched, if it was first on the "market", and to dominate even after its accuracy has been surpassed.

A classic example of the expedients we sometimes use in modelling is the use of vorticity instead of strain in production terms [27]. This step is neutral in thin shear flows, since both reduce to the shear rate, but it solves the long-standing problem of excessive turbulence levels in the approach to stagnation points. In two-equation models, using vorticity is not legitimate, because the exact production terms contains the strain rate instead. Typically, vorticity is used as a "temporary" expedient, which does nothing for the implicit hope that the dependence on empiricism will gradually decrease. The quandary seems intact as of 1998 [6].

There is little dispute that the ultimate potential of eddy-viscosity models does not include separated flows over 3D geometries. This statement is widely accepted for calculations in steady mode, on the argument that the coherent shedding motion is very different from that in thin shear flows. Calculations in unsteady mode have more potential but still have severe limitations, as discussed in §2.4. Two-dimensional bluff bodies are sufficient to make the simple models fail even with sharp corners, which set the separation location. The models are just too simple and replete with empiricism, and are trained in such a small pool of simple shear flows, that they have no deep reason to generalize to complex flows. We should however heed a remark of Hunt [23], here slightly paraphrased: "It is important to note that in most flows (including those over bluff bodies) where the duration of a distortion is smaller than the intrinsic time scale of the turbulence, there is insufficient time for the turbulence to affect the *mean* flow and therefore an erroneous turbulence model has little effect on the *mean* flow. Thus, fortuitously, in most turbulent flows one-point models of turbulence only affect the mean flow calculations where the models are most appropriate (namely in shear flows where the intrinsic time scale is smaller than the distortion time scale)".

Hunt appears to place all one-point RANS models, of any complexity, in the class of turbulence treatments that have "no reasonable claim" to provide accurate stresses in complex flows, but in some useful situations do "little enough damage". The assessment is harsh, and high-quality modelling work has been dedicated to proving it unfair. However it has a basis, and some of us can accept it for of their own models, particularly if they are simple.

An example is given by Ying *et al.*, who compare measured and calculated Reynolds stresses over a multi-element airfoil [58]. This is the type of flow Hunt had in mind. As the shear layers (boundary layer and slat wake) pass over the trailing edge of the main airfoil element, the streamlines have a modest deflection, as part of the abrupt merging with the stream from below the trailing edge. The strain-rate tensor has an excursion that is *not* modest, and propagates to the computed Reynolds-stress tensor. A transport-equation eddy-viscosity model was used. In contrast the measured Reynolds stresses show no such excursion, and their behavior is consistent with that of a "conserved quantity", with only a slow evolution in the streamwise direction. The anomaly in the computed stresses is due to the eddy-viscosity approach (the eddy viscosity is conserved, instead of the stresses). It certainly makes the experiment-computation comparison more delicate: the agreement level varies drastically within a short distance. On the other hand, the velocity profiles downstream show no clear sequel of the stress excursion, as predicted by Hunt.

Another wording of Hunt's line of thought is that quite a few flow regions that appear complex and/or 3D are shaped by "vortical inviscid" physics, and not by the *local* turbulent stresses. It may be the case for the "necklace" vortex at a wing-wall junction. Its characteristics may depend more on the upstream growth of the boundary layer, which a simple model can accurately predict, than on the Reynolds stresses inside it. This contrasts with the secondary vortices in a square duct, which are created by the turbulence. These vortices expose linear eddy-viscosity models, but their practical importance is modest. Thus, a simple model can "get credit" for the successful calculation of a new flow, merely because the Reynolds stresses it generates in the complex regions are not damaging; usually, it is just as well if the stresses are too weak. The chances that the flow feature will be smeared due to insufficient grid resolution are also higher in such regions, because the user's experience base or willingness to manually refine the grid is less than in attached boundary layers. Unstructured adaptive grids will address that problem, but only in the next generation of codes.

Hunt's optimism does not extend to bluff-body flows such as a stalled airfoil, because now the intrinsic time scale of the turbulence is the same as that of the distortion. In view of their limited prospects after separation, it is natural that most of the refinements applied to simple models will be aimed at their accuracy in boundary layers, including short separation bubbles, curvature, compressibility, and a few thin shear flows. This is Challenge I.

2.3. Complex RANS models

The simpler transport models will remain useful and receive slight improvements, but the state of aeronautical CFD makes difficult to evade the conclusion that a decisive improvement in turbulence accuracy must be achieved before CFD becomes general. It is a matter of debate whether higher-quality models will provide that answer. The primary candidates are Reynolds-Stress-Transport (RST) models. Now these models have a much closer connection to the equations, and boast several exact terms. An RST model would remove the anomaly noted in §2.2 with the sudden distortion over the trailing edge. With proper attention to invariance, RST models *should* generalise from their "training ground" to flows with curvature, or vortex and similar flows, much more reliably than eddy-viscosity models. On the other hand, they also contain many empirical terms particularly in the pressure and dissipation areas, adjusted by trial and error. For some of these quantities, data can be obtained only from DNS which has been limited to simple geometries, although progress is being made. In addition, precise term-by-term matching is often too much to ask for; compensating errors, for instance between the anisotropy of the dissipation and pressure-strain tensors, appear both common and acceptable.

In terms of the two challenges, RST models have a tentative advantage over simple models for Challenge II, the separated and vortical regions [28]. They are often far from "user-friendly" in the sense of Challenge Zero. For Challenge I, incipient separation, no model can succeed without excellent empiricism, and it is no easier to impress such empiricism on a complex model than on a simple one [29]. In fact the exact character of certain terms puts them off limits to empiricism; in that sense, RST models are more difficult to "steer".

Assessing true progress is made difficult by the constant modifications made in publications; sometimes the reader cannot be sure that the new version of the model also succeeds where the last version did. Another concern is the persistence of controversies such as about the use of "wall-reflection" terms, or the question of whether RST models reproduce curvature effects without additional empirical modifications. Similarly, Zeman's study of free vortices implies that even RST models need specific curvature/rotation modifications to reproduce the damping of the turbulence [59]. Not only does this make the hope of an elegant resolution to Challenge II seem very remote, but streamline curvature is not a Galilean invariant [51], and therefore Zeman's model for that flow is not application-ready. Separated cases which are problematic for simple models, for instance strong shock interactions, are also problematic with complex models [21]. Possibly, CFD solutions with any model suffer from numerical errors in strong interactions.

At the risk of minimizing the work of fellow modellers, the author deems it unlikely that a RANS model, even complex and costly, will provide the accuracy needed in the variety of separated and vortical flows we need to predict. The intellectual task of synthesizing a large body of available findings into a truly higher and durable version of a complex model is huge, and few model developers seem keen on doing it. Large groups tend to publish along "tentacles". This fits better with educational, institutional and funding needs than with the needs of the code writer who is searching for a robust, stable and understood formulation.

After studying the turbulence-modelling field, the author's colleagues in CFD at Boeing again and again have asked him for a "first-principle-based" turbulence model. So much empiricism makes the approach appear highly unreliable. It is more than plausible that Reynolds averaging suppresses too much information, and that the only recourse is to renounce it to some extent, which means calculating at least the largest eddies simply for their nonlinear interaction with the mean flow. This step appears desperate to observers, especially the mathematically oriented ones, with some reason. Prof. Jameson remarked that "we should not compute 1-centimetre eddies over a Boeing 747".

2.4. URANS

The first alternative to complex RANS modelling has been called "Very-Large Eddy Simulation" (VLES) or "Unsteady Reynolds-Averaged Navier-Stokes" (URANS). The URANS acronym is more descriptive, and will be used in this paper. Such calculations rely on traditional RANS models but are deliberately unsteady, even with steady boundary conditions. For instance, vortex shedding is allowed past a bluff body [24,38,44,6]. Durbin correctly noted that the Reynolds stresses created by the time averaging of the URANS solution overwhelm those carried by the model itself, in the separated region, and therefore remove much "responsibility" from that model [14]. Nixon's group used the acronym VLES for some very interesting work [10], but it should be classified as LES, and certainly

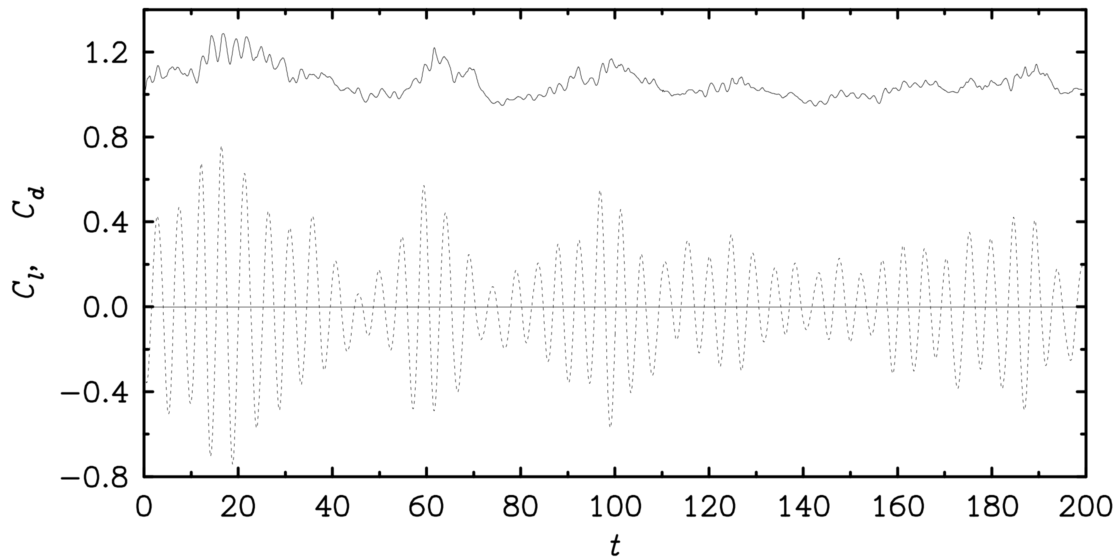

Figure 2. Lift and drag coefficients for circular cylinder. DES at $Re = 5 \times 10^4$ [55]. Averaged over 2 diameters spanwise. —, C_d; - - -, C_l.

not as URANS. This is another reason to avoid the VLES notation.

It is simple to describe how to conduct a URANS, but the physical implications give pause. For simplicity, consider vortex shedding by a bluff body and assume that URANS gives a periodic solution, which is typical [14,44,36,55]. We are therefore envisioning a flow field that is periodic in time and smoother than the true turbulent field but representative in some sense, so that its inviscid dynamics mirror the large-scale dynamics of the true flow. Such a field can be defined from the full turbulent one by phase-averaging. Plausible references can be found to adjust the phase slightly at every cycle. However this ignores the amplitude modulations, which are strong for bluff bodies and have overwhelming experimental and simulation evidence [36,55]. Figure 2 shows results for a circular cylinder. The peak lift coefficient, five cycles apart in the same flow, can easily vary by a factor of 2. As an analogy, this type of averaging would amount to averaging a number of human beings that walk by and do not have the same height. The average is not a human body. Returning to the flows, the phase-averaging creates Reynolds stresses of two kinds, and in URANS both are left to the turbulence model. The expected kind, due to incoherent small-scale motions, appears amenable to common modelling. The troublesome kind is that due to modulations in the shedding. The difference between fields at the same phase in different cycles is just as coherent as the difference bewteen these fields and the long-term time average. It is just as difficult to model with the simple common approximations. Fortunately, it is smaller. Nevertheless, it is not rigorous to state that, with URANS, the turbulence model is responsible only for "incoherent" motions. Furthermore, the interpretation that only random and presumably weak (in terms of velocity and/or time scale) turbulence is appropriate for modeling is inconsistent with the simple fact that the models are usually adjusted to reproduce the spreading rate of mixing layers *in RANS mode*. Yet the mixing layer is notorious for its coherent vortices, which span its whole width and pair in obvious discrete events.

Another slant is to observe that URANS implies a separation of time scales, between that of the visible shedding and that of the putative residual turbulence, which is not indicated in measurements of spectra for instance with hot wires. Nevertheless, the approach has plausibility, and certainly improves results quantitatively for bluff bodies; the shedding frequency is often excellent. In the boundary layers, there is no strong additional reason to distrust the models, which operate in quasi-steady mode. Also note that for bluff bodies, conducting unsteady calculations is optional *only* in somewhat artificial conditions: those in which a steady solution can be obtained by imposing symmetry, or a large time step, or using a Newton method for convergence. More frequently, a user that is after massive separation will find that the code simply cannot find a steady state, and that the only course is to operate in a time-accurate mode and analyze an unsteady solution.

A 2D URANS recognizes the role of time, but not of the spanwise coordinate z. It has now been established by many simulations of bluff bodies that ignoring the z-dependence is *not* safe [26,35,31,25]. At least in LES and DNS, 3D solutions in 2D geometries are highly justified. They are denoted by "3/2D" in the table by §3. A z-dependence obviously belongs in a thorough study at the URANS and higher levels, but its role is clouded by several facts. The spanwise boundary conditions are arbitrary; periodic conditions are common and plausible, but some studies use reflection conditions at the side boundaries. The size of the spanwise domain is also arbitrary. Systematic tests of that parameter are costly. This issue will resolve itself in practice, in the sense that actual geometries are 3D, but it is an obstacle to a simple understanding of the nominally 2D flow, and to efficient validations of the various approaches.

2.5. LES and DES

Away from boundaries and without chemistry, Large-Eddy Simulation appears well understood, and in this author's opinion there is little to gain by refining the SGS models [16]. Some proposals appear to add mathematical rigor, but amount to accounting changes. Others use sensible approximations, but fail to show a clear advantage over the *same-cost* LES with a simplistic model, which would have a slightly finer grid. Issues such as non-commutativity between the filtering and differencing operators are valid, but their leverage is probably small compared with SGS modelling errors. It is even possible to run an LES without any SGS model, simply using an upwind or monotone numerical scheme to offset the energy cascade and maintain the smoothness of the solution. Such an exercise is not greeted warmly by those who built careers on SGS modelling, but it is far from absurd [17]. The procedure may damage a somewhat wider band of the spectrum than a purpose-designed SGS model does (much like the difference between a simple viscosity and a hyper-viscosity in homogeneous turbulence), but a large enough grid will produce an inertial range. Galilean invariance is broken by the asymmetric schemes (with a centered scheme, time-integration errors are also non-Galilean-invariant, but they are weaker than here, when they amount to an SGS model). A much more serious concern is that SGS-free LES cannot deal with the wall region of the boundary layer (except by approaching DNS).

In the wall regions, it is fair to describe most of the current LES work as Quasi-Direct Numerical Simulation (QDNS) [49], for three reasons. These simulations resolve the near-wall "streaks", thus leaving no eddy type unresolved. The SGS stresses are of the same order of magnitude as the viscous stresses, since typical eddy-viscosity levels are very close to the molecular viscosity. Finally, the grid spacing in all three directions is clearly limited in wall units; the streamwise spacing may rise from 20 in DNS to 50 in QDNS, at best. Typically, the saving in computer time is a factor of 10, roughly equivalent to a Reynolds-number increase by the modest factor $10^{1/4}$; this is hardly worth the empiricism. An accurate and reliable "true" or "full" LES, meaning that the Reynolds number based on the grid spacing is unlimited, appears to be a difficult goal. Presumably the community switched to QDNS, although the pioneering work [13,42] was full LES, in order to reduce empiricism in the near-wall treatment. In full LES the grid spacing in all directions (or at least in the two directions parallel to the wall) would scale with the boundary-layer thickness, for a given level of accuracy. In this area, huge gains are expected from improved SGS modelling. However, it is *unavoidable* that empiricism will be added. At the least, such a treatment would have to imply values for the constants in the logarithmic law. This is a key statement, which needs to be pondered and challenged, and could force useful debate. It is a normal desire to reduce the empiricism, and agreeing on any hard limits in that process will organise our thinking.

The full-LES method reverts to quasi-steady RANS behaviour very near the wall. It is quasi-steady in the sense that the time scale of resolved variations is much larger than the internal time scale of the turbulence model, which scales with the inverse of the shear rate. Typical values at the start of the logarithmic layer could be the following, all in wall units: the wall-parallel grid spacing is 1000, so that the minimum size of a flow structure is 3000; its convection velocity is of the order of 15; therefore the passage time is 200; the shear rate is about 0.1, much larger than the inverse of the passage time. It is RANS in the sense that the model is very similar to pure RANS models, and that grid refinement does not eliminate the influence of the SGS treatment rapidly at all (until an extreme refinement turns the method into QDNS).

A robust and sufficiently accurate treatment of that kind is a plausible target. The streaks seem to be nearly as "numerous and universal" (and therefore amenable to modelling) as the small eddies in free turbulence (the streaks are not isotropic, but calling the small eddies of free turbulence isotropic is misleading: the collection of eddies that are modelled as SGS in one grid cell at one time step obviously has preferred directions). It is only that the streaks have much more leverage than the small Kolmogorov eddies. It will be well worth the effort, for several reasons. First, the law of the wall is quite a robust feature of boundary layers, although we expect an erosion of its domain of validity, expressed as y/δ, in strongly stimulated flows such as by a pressure gradient (δ is the boundary-layer thickness). We also expect an erosion from a reduction of the statistical sample. Dr. G. Coleman and the author explored such a "local log law" in channel-flow DNS, with mixed results (unpublished). An oobstacle is that the DNS domain size is not very large. The pressure-gradient erosion can be addressed by a finer grid which will resolve, instead of modelling, smaller eddies closer to the wall. Second, in 3D flows, the mean skin-friction vector is very unlikely to vanish; thus, the law of the wall *could* retain its validity even under a separating boundary layer. Finally, most of the difficulty in RANS modelling of strongly stimulated boundary layers resides in the outer region. There, LES can clearly capture effects such as straining, cross-flow, and curvature. Therefore, LES addresses both Challenges, I and II. However, it is at a considerable cost over RANS. In addition, wall-bounded LES with the streaks modelled can be described as hybridized with RANS, although the implied empiricism is confined to a shallow layer.

At a recent LES workshop, a variety of QDNS and full-LES methods were applied to fairly simple geometries with sharp corners [41]. In spite of these helpful features, the conclusions were particularly mixed, and did not make LES or even QDNS appear very mature. The flow past a circular cylinder, even at Reynolds numbers of a few thousand, has also led to quite different levels of agreement with experiment in the last few years. For instance Breuer was disappointed with the results of both SGS-model improvements, and grid refinement [7]. We also know from personal communications of at least two studies which their authors did not consider successful enough to publish. SGS models also remain quite different between different schools, again suggesting a lack of maturity and/or indifferent progress.

§3 re-iterates that LES, even with the best wall-region treatment, is very far from affordable in aerodynamic calculations, and will be for decades [49]. This is due to the large regions of very thin boundary layers, where δ is of the order of 0.1% of the airfoil chord c. It led us to propose Detached-Eddy Simulation (DES), a further step in the hybridization of LES [49]. The idea is to entrust the whole boundary layer (populated with "attached" eddies) to a RANS model, and only separated regions ("detached" eddies) to LES. It is aimed primarily at external flows. It is consistent with the two positions that Challenge I is a reasonable one for RANS models whereas Challenge II is not, and that LES is well understood away from walls. We show below that it leads to a manageable computing cost even at high Reynolds numbers [43].

A typical application of DES is to a wing with a spoiler or a landing gear. Large areas of boundary layers are treated efficiently with quasi-steady RANS (the time scale of the global unsteadiness of the flow, which these boundary layers experience, is large compared with the inverse of the shear rate inside them). The model accounts for all the turbulent stresses across the whole boundary layer, just like in a simple RANS run. Behind the spoiler, the momentum transfer is dominated by large unsteady eddies which are candidates for LES on two counts. First, they are not as numerous as the "horseshoe" vortices in the outer part of a boundary layer (let alone the wall streaks) and second, they are geometry-specific. An additional benefit of DES is its unsteady information. Though useless for many purposes, such as the range of the airplane, it will sooner or later be of great use for structural or noise studies.

An attractive feature of DES is that it is simply formulated, and already being tested. This is not the case for similar concepts which have been informally envisioned. Many of them are zonal. DES is not, which is much preferable for routine use, and only requires a quick alteration of the S-A one-equation model. On the other hand deriving an efficient unsteady code, as needed for DES, from a steady one is not trivial. DES results for an airfoil at high angles of attack, a classical Challenge II example, have been presented [43] and fig. 3 is reproduced from that paper. At high angle of attack the agreement with experiment is surprisingly good, but no better than in the best examples of DNS and LES for bluff-body flows [35,31] (at lower Reynolds numbers). In addition, the finding that 2D simulations produce an excess of drag is consistent across studies. At low angles of attack the simulations correctly reduce to RANS; these flows are out of reach of DNS

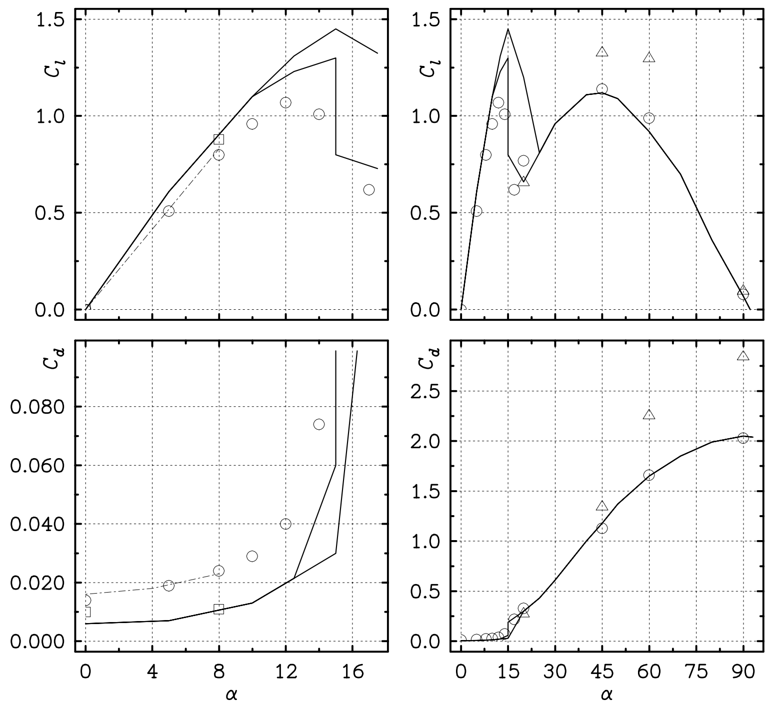

Figure 3. Lift and drag coefficients for NACA 0012 airfoil [43]. —, Exp. $R_c > 10^6$; - — -, Exp. $R_c \simeq 10^5$; ○, DES $R_c = 10^5$; □, DES $R_c = 3 \times 10^6$; △, URANS $R_c = 10^5$.

and LES. DES will require definite skills from the user in directing the grid resolution; however, RANS also benefits from careful grid generation. Presently, a few patient RANS users are refining grids "manually" after exploring preliminary solutions [58], but it can be feared that many solutions are under-resolved in the separated regions.

Figure 4 illustrates the difference in description for the flow past a circular cylinder. The runs on the right are DES, but the visible eddies are in the LES region. Of course the DES figures show only a plane out of the 3D domain. Each step from SRANS through 2D URANS to LES/DES adds a dimension: first the time and then the third space dimension. The cost increase is of an order of magnitude each time. In return, the SRANS drag is too low at $C_d \approx 0.9$, the URANS drag is much too high at $C_d \approx 1.7$, and the DES drag is in much better agreement with experiment, although grid effects are still present: $C_d = 1.05$ on the coarse grid but 1.36 on the fine grid [55]. The experiment gives 1.2. The DES on two grids also depicts the inclusion of smaller eddies allowed by the finer grid. The flow is at a Reynolds number of 50,000. It is modelled with laminar separation, in the following manner. By setting the inflow condition for the turbulence variable to zero, we have no eddy viscosity in the attached boundary layers. Nonzero turbulence values are injected initially, and contaminate the shear layer slightly downstream of separation, so that the model remains active in the recirculation region only, after losing memory of the details of the initial condition. We can also obtain turbulent boundary layers, with an appropriate nonzero inflow value (thus triggering the drag crisis of the cylinder [55]); the key is to selectively set the inflow and the initial conditions. This decision is made by the user outside DES itself, and outside the turbulence model, which cannot predict transition due to boundary-layer instabilities. Since the boundary layers are laminar, there is little

Figure 4. Simulation of flow past circular cylinder by various approaches [44,55].

difference between DES and LES at this Reynolds number.

Some similarities between DES and past treatments of the wall region in LES [13,42,4] have led to comments such as "DES contains nothing new". The similarities are not in the details (which are vastly different between methods) but merely in the position that the near-wall flow field, averaged over a grid cell, behaves closely enough to a full Reynolds-averaged flow that the law of the wall and/or RANS modelling technology are good approximations. These comments stem from a narrow focus on the historic applications of LES, such as channel flow. There, it is correct that DES is no more or less plausible than methods which blend simple buffer-layer models and simple SGS models. Channel flow will be discussed shortly. For instance, one could well use an eddy viscosity that is the smaller of the one given by the mixing-length approximation, with Van-Driest damping, and the Smagorinsky model. However the additional capability of DES, relative to all these methods, is clear: to treat the entire boundary layer in RANS mode. A mixing-length model does not have this capability; the lowest level that does is an algebraic model such as Cebeci-Smith. Algebraic models do not lend themselves to complex geometries, unstructured grids, or to function under detached eddies; therefore, one-equation models are the simplest type that make DES practical. Since their accuracy is acceptable in boundary layers, DES is possible now with reasonable ambitions of accuracy, for instance over a sphere or cylinder.

DES was tested in a channel, with LES grids, by three groups [37]. The approach was not adjusted at all for this flow, the grid spacings parallel to the wall were several hundred wall units. In such a simulation, DES relaxes the restrictions on the wall-parallel spacing, but not the wall-normal spacing which has to be of the order of $y^+ = 1$. We obtained stable and reasonable results, but the additive constant C in the logarithmic law is not very accurate due to a "buffer layer" between the region in which the stress is modelled and the region in which it is resolved. The results of Wasistho and Squires at a friction Reynolds number $Re_\tau = 2000$ are shown in fig. 5. The coarser grid has $64 \times 64 \times 32$ points and the finer one $128 \times 128 \times 64$ points, both in a domain of size $2\pi \times 2 \times \pi$. The velocity profiles display the shift between modelled log layer and upper layer. The finer grid does not strongly improve the shift, which is associated with the current calibration of the SGS model in DES. The shear-stress profiles display a large transfer from the modelled shear stress $\overline{\nu_t(u_y + v_x)}$ to the resolved stress $-\overline{uv}$ when the grid resolution is doubled. Therefore, the two cases are deeply different, but the mean velocity changes little. The viscous stress $\nu\overline{u_y}$ is almost identical on the two grids. These results suggest that DES has potential as an approach to wall modelling, even though such an application was not designed for and is not natural.

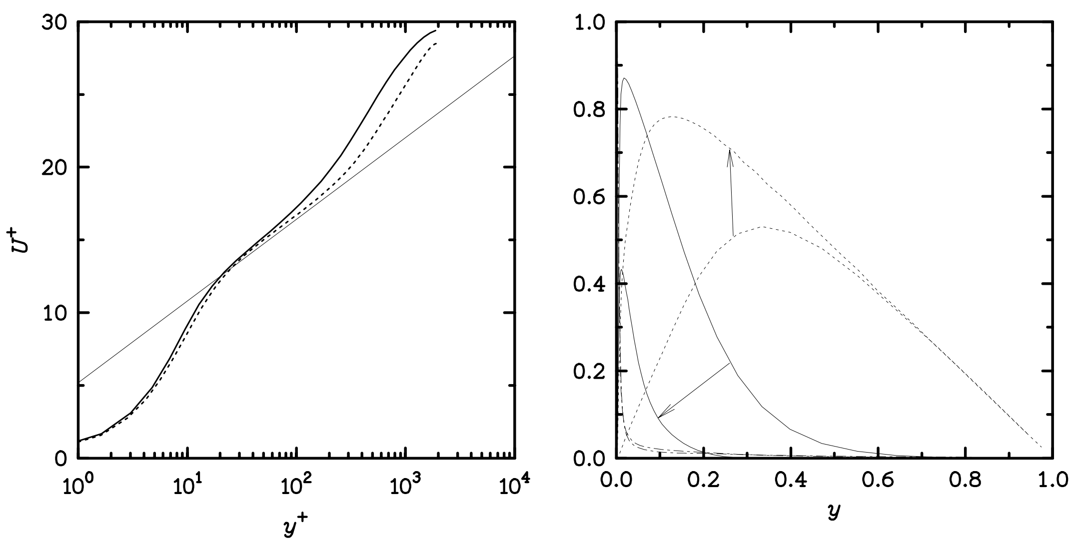

Figure 5. Channel DES by Wasistho & Squires (1999). Left, velocity; —, coarse grid; - - -, fine grid. Right, shear stresses. - - -, resolved; —, modelled; - —, viscous. Arrows point from coarse-grid to fine-grid results.

Still another hybrid concept can be formulated. It is zonal and would consist in treating the "unchallenging" regions of the boundary layer with RANS, and the Challenge-I regions with full LES. The method would switch to LES upstream of any intense pressure gradients, which tentatively makes another step up in accuracy. The difficulties are: the "artificial intelligence" of identifying where the switch should be located; the generation of quality turbulent eddies for the RANS region to dispatch into the LES region (the regions might have to overlap). This is a concept that would live much more easily in a 2D boundary layer than on a 3D object. The "eddy seeding" problem appears less severe with DES, because a separated shear layer is exposed to vigorous new instabilities, thanks to both the removal of the wall confinement and what can be loosely called "absolute instabilities" [22] (in contrast, the RANS-to-LES switch in the other concept would occur in a region of "convective instabilities"). These new instabilities plausibly overwhelm the boundary-layer turbulence. If they do not, for instance in a separation bubble, DES cannot be expected to provide much improvement over RANS.

2.6. DNS

The value and requirements of Direct Numerical Simulation are well known. Few DNS projects have been conducted at a "full" Reynolds number, but the attachment line of swept wings is an example [46]. DNS was applied at the (local) Reynolds number of the flow on an airliner. This is a case of "microscopic" simulation, in which it is justified to isolate a very small region of the flow (the justification relies on experiments). The author once received dubious praise for simulating "a milli-second over a postage stamp"! Simulations of homogeneous turbulence and of other boundary layers could also be described as microscopic. DNS of a whole device is normally out of the question. It is a beautiful research tool; in fact in the author's opinion its reach is sometimes under-estimated, due to a misguided insistence on simulating at the "right" Reynolds number.

The argument, which has long been a minority one, is the following. When asking a fundamental question in turbulence, assume we have the choice between a DNS and an LES having the same cost. The DNS will have a slightly larger range of scales in each direction (certainly less than double), because of saving the SGS-model cost. The LES will have a higher Reynolds number; if a QDNS, the difference will be less than a doubling. The LES will assume that the unresolved eddies have a simple enough behaviour to be modelled; for instance the great majority of the Reynolds shear stress will be resolved, at least in the challenging region of the domain. If so, the same-cost DNS can run at a Reynolds number sufficient to sustain turbulence. Extrapolating the DNS results to the LES Reynolds number can *also* be done with confidence (especially if the DNS is possible over a range of Reynolds numbers). An extrapolation "after the simulation" is inexpensive and can be refined, much more easily than the LES can be re-run with a different SGS model (or adjustable constants). By that standard, we could have counted one run for a thorough DNS study, as opposed to maybe three runs for a thorough LES study, which changes the cost balance somewhat. Three to four runs is typical in LES studies, many of which are presented as comparative tests of SGS models and/or as tests of LES itself, by comparison with experiments. In contrast, in the author's opinion, a DNS study can be free-standing [52]. In addition, the extrapolation can then reach any Reynolds number (this amounts to the view that turbulence is more predictable, the higher the Reynolds number, which is not shared by all). Atmospheric scientists never consider DNS for the Planetary Boundary Layer, but fundamental PBL questions can very well be asked with DNS and extrapolation [11], and the anti-DNS attitude is counter-productive.

One good reason for doing QDNS is the comparison with a laboratory experiment that is moderately out of reach of DNS. This occurs typically when the experiment was designed to allow measurements of the smallest eddies; physical limits restrict the possible range of scales, but not as severely as DNS does.

Below, DNS is classified as requiring "no empiricism". This does not imply that the DNS of a complex flow is free of decisions even once an accurate DNS code has been created. In the case of channel flow, the decisions consist in the grid spacing, time step and domain size. For these, the direction of "goodness" is clear: smaller spacing and larger domain. Homogeneous turbulence adds the influence of the initial conditions or stirring system, for which goodness is not simply a direction; there is an art. Flows containing transition require many further decisions, regarding the freestream-disturbance and wall-imperfection content and the vibrations. This fine information is not found in the CAD file of an airplane or car. Engineering DNS would not be a "black box".

Recent Reynolds-number increases in DNS have been modest, partly because the "super-computers" have nearly stagnated, certainly compared with personal computers (more accurately, the DNS share of the super-computers has nearly stagnated). The channel has now reached $Re_\tau = 590$ [33] but the boundary layer had reached $Re_\tau = 650$, with a more ample horizontal domain, in 1988 [47]. For a really attractive new study, for instance to lock the value of the Kàrmàn constant, a factor of 5 or preferably 10 in Reynolds number over the current highest would be needed. Therefore the DNS effort has, correctly, be directed instead at simulating more complex geometries, or simple ones with strong pressure gradients, three-dimensionality, rotation and curvature, complex deformations, heat transfer, combustion, shock waves, and noise [2,1,12,18,34,40,57]. Fully successful DNS studies of the supersonic boundary layer should appear very soon. The current standard includes "reasonable results" but not quantitative comparisons, a problem being that low-Reynolds-number supersonic experiments are lacking [1]. A definitive study of the interaction with a normal shock will certainly be of great interest to the airliner industry.

2.7. Role of grid refinement

In RANS, the equations possess a smooth exact solution, and the numerical solution approaches that solution as we refine the grid. The aim of grid refinement is numerical accuracy. In contrast, in LES as it is practised and in DES, the Sub-Grid-Scale (SGS) model adjusts to the grid so that the smallest resolved eddies match the grid spacing. Recall fig. 4 for the visual aspect and fig. 5 for the corresponding stresses. In a finer grid, resolving eddies to a smaller size gives the large energy-containing eddies more eddies for genuine nonlinear interactions, making them more accurate. The aim of grid refinement is now *physical* instead of *numerical*, to use simple words. This distinction is tracked in the table in §3 (several methods had to be labeled "hybrid", as their aim is different in different flow regions). Another description of it is that when the aim is numerical, the turbulence model does not depend on the grid spacing but when the aim is physical, it does. A consequence is that in URANS, no amount of grid refinement will override the influence of the empirical content of the turbulence model. In contrast, in a method with the "physical" aim, grid refinement weakens the role of the modelled eddies and thus improves the fidelity of the simulation. A 20% change of the Smagorinsky constant in a well-resolved LES is minor, but a 20% change in the Kàrmàn constant is not.

It has been proposed *not* to automatically link the width of the LES filter and the grid spacing, in order to obtain solutions of the filtered equations free of significant numerical errors. A typical procedure to adjust an SGS model has been to seek a –5/3 slope for the energy spectrum all the way to the cut-off wave-number of the grid. Then, the numerical errors remain the same fraction of the SGS kinetic energy, and it is fair to write that "numerical and SGS effects cannot be separated". This situation is disturbing to some careful people, who would prefer to understand the physical system of the filtered equations, and then obtain very accurate solutions to it. That seems possible only if the spectrum rolls down from its –5/3 slope some distance from the cut-off. The author's opinion, which is not based on tests, is that this would not be the best use of numerical effort. Widening the filter raises the magnitude of the sub-grid stresses, which are notoriously inaccurate locally. It is unlikely that this effect can be offset by the reduction of numerical errors, or even by the design of a superior SGS model, possibly gained from the better physical understanding. On a given grid, an LES with a wide filter would cost almost the same as one with a narrow filter: the cost per step would be the same, and the time step could only rise by a modest amount. Conclusive tests of the filter-grid relationship would crucially depend on the definition of a figure of merit.

3. COST ASPECTS

Here the aim is a broad view of the methods with the order of magnitude of their cost, translated into a readiness date. Such predictions are not wihtout risk. Time will tell how far off these dates are, but unless they are hugely in error, they are valuable for research planning. The principal definitions and assumptions which entered the estimates in Table 1 are the following. The target flow is that over an airliner or a car. The acronyms in Column 1 have all been used above. "Aim" in Column 2 refers to the aim of grid refinement: numerical, or physical (§2.7). The Reynolds-number dependence refers to the number of grid points. The step from "strong" to "weak" Reynolds-number dependence indicates a change from a slow logarithmic dependence similar to that of the skin-friction coefficient, to a strong one similar to that of the viscous-sublayer thickness. "3/2D" refers to simulations which are 3D even if the geometry is 2D. When the geometry is already 3D, 3/2D means that the grid spacing scales with the shorter dimension of the device, and does not "take advantage" of high aspect ratios. A clear example would be a wing flap: a 3DRANS will cluster points near its tips but use a loose spacing elsewhere, whereas a 3/2D method will space points by the same fraction of the flap chord all along. The step from "strong" to "weak" empiricism indicates, perhaps arbitrarily, that the only remaining adjustable constants are those in the Law of the Wall.

For the grid spacing, RANS and DES figures are based on current practice. The requirements are well understood for the spacing normal to the wall. In the other directions, the geometry is assumed to have only a moderate number of features such as flaps and spoilers. Under "DES" are included both strict DES as defined in [49], and other hy-

Table 1
Summary of strategies

Name	Aim	Unst.	*Re*-dep.	3/2D	Empiricism	Grid	Steps	Ready
2DURANS	num	yes	weak	no	strong	10^5	$10^{3.5}$	1980
3DRANS	num	no	weak	no	strong	10^7	10^3	1990
3DURANS	num	yes	weak	no	strong	10^7	$10^{3.5}$	1995
DES	hybr	yes	weak	yes	strong	10^8	10^4	2000
LES	hybr	yes	weak	yes	weak	$10^{11.5}$	$10^{6.7}$	2045
QDNS	phys	yes	strong	yes	weak	10^{15}	$10^{7.3}$	2070
DNS	num	yes	strong	yes	none	10^{16}	$10^{7.7}$	2080

brid methods which are likely to be developed in the next few years, with the general expectation that they will treat the simple attached boundary layers with RANS. For such methods, a grid block of the order of 64^3 points appears adequate to resolve a separated region, since about this many points were used on the stalled airfoil [43]. At higher Reynolds numbers, the cost increase is modest, since only the normal grid spacing needs to be reduced. Thus, the grid increase over 3DRANS is plausibly in the millions of points, not tens of millions, and 10^8 is fair for the grid count. For LES we had estimated 10^{11} for a clean wing [49], leading to a few times more for the whole aircraft. The DNS estimate is based on grid patches with an area $\Delta x^+ \times \Delta z^+$ of 100 wall units and a chord Reynolds number for the wing of about 7×10^7, and agrees with that of Moin & Kim [32]. Estimates would be lower for a road vehicle, which has a lower Reynolds number, but the difference would be a large one only for QDNS and DNS, with their strong cost dependence. The number of steps uses the same grid information and CFL numbers of order 1, if unsteady, and assumes the simulation needs roughly 6 spans of travel for an airplane.

The readiness estimates are based on the "rule of thumb" that computer power increases by a factor of 5 every 5 years. This will be disputed, but another rule has been a factor of 2 every 2 years, which is not much faster. Readers are free to apply their favorite rate of progress, starting from the assumption that a very expensive problem today costs about 10^{15} floating-point operations. "Readiness" roughly means that a simulation is possible as a so-called "Grand Challenge". Industrial everyday use will come later. Dates are rounded to the nearest 5 years.

4. OUTLOOK

Progress in numerical methods and computers is intensifying the challenge for turbulence treatments, to provide a useful level of accuracy in slightly or massively separated flows over fairly complex geometries at very high Reynolds numbers. This is desirable in the near future, between 5 and 10 years, and not only on a research basis; industry is more than ready for this capability, especially the jet-engine industry. In addition, the needs of non-specialist users and automatic optimizers dictate a very high robustness. Flows with shallow or no separation appear to be within the reach of the current steady RANS methods or their finely calibrated derivatives, incorporating modest improvements such as nonlinear constitutive relations. For such flows, transition prediction with generality, accuracy, and robustness may prove more challenging than turbulence prediction.

With massive separation, it appears possible we will give up RANS, steady or unsteady. This will probably be the major debate of the next few years. The alternative is a derivative of LES, in which the largest, unsteady, geometry-dependent eddies are simulated and (for most purposes) "discarded" by an averaging process. We have to balance our ambitions with cost considerations, and a table summarizing the issue was tentatively provided. A major consideration is whether LES is practical for the entire boundary layer, and it was strongly argued that this will not be the case, in the foreseeable future. This forces hybrid methods, with quasi-steady RANS in the boundary layer. In this paper, LES was effectively defined as a simulation in which the turbulence model is tuned to the grid spacing, and RANS as the opposite. Other more subtle definitions probably exist, but this one seems to classify almost all the studies to date. Speziale's hybrid proposal involves the grid spacing and the Kolmogorov length scale but, surprisingly, not the internal length scale of the RANS turbulence model; thus, it is difficult to classify [54]. The proposal of Aubrun *et al.* is also hybrid, as it leads to combining "modelled" and "resolved" Reynolds stresses, but the modelled stresses do not scale (and vanish) with the grid spacing as they do in LES [3]. Variations on the now-running DES proposal clearly have a wide window of opportunity.

The plausible spread of hybrid methods highlights the permanence of a partnership between empiricism and numerical power in turbulence prediction at full-size Reynolds numbers. This demands a balance in funding and in publication space. Since hybrid methods offer leeway when setting the boundary between "RANS regions" and "LES regions", the more capable the RANS component is, the lower the cost of the hybrid calculation will be. Therefore, the switch to LES in some regions does not remove the incentive to further the RANS technology. This scene also raises the issue of which core of experiments and DNS will be the foundation of the empirical component of the system. As ever, we will need simple flows for calibration of the RANS sub-system, and more complex flows for validation of the full CFD system.

Acknowledgements

The author is grateful for the comments of J. Crouch, W-H. Jou, G. Miller, A. Secundov, K. Squires and M. Strelets, and for years of informal teaching by P. Bradshaw.

Appendix: the Spalart-Allmaras model

We give a little background on the S-A model, since it appears several times in the paper. The model was inspired by the work of Baldwin and Barth, who invited the author to critique their model [5]. In both cases the principle was to create a model that was complete in the sense of Wilcox [56], which essentially meant it would be a transport model instead of algebraic, and as simple and numerically undemanding as possible. No deep reason was seen why two equations were indispensable, although this remains a widespread position, and a high reliance on intuition was accepted. Connections to exact transport equations were not sought, other than the obvious invariance requirements.

A crucial term in the S-A model is the wall destruction term, which we found was identical to that of Secundov from 1973 [20,56]. The term depends on the wall distance d, which has a numerical cost and is considered undesirable by some colleagues on physical grounds. We disagree with this, and believe the term sensibly mimics the confinement of the eddies, expressed by the pressure term in the transport equation for Reynolds shear stress. The viscous sublayer treatment borrows from Baldwin-Barth and is aimed at low cost. The production term is proportional to vorticity, instead of strain rate as for the turbulent kinetic energy; this is another untuitive choice. Similarly, the diffusion term is not conservative for the eddy viscosity; instead it conserves the eddy viscosity raised to the power 1.622.

The model was studied in depth in terms of Challenge Zero, which defeats the B-B model (ultimately because in B-B the destruction term is based on a gradient instead of the wall distance). It was calibrated on Challenge-I cases, using a subset of thin shear flows that could be handled with a few adjustable constants and were closest to the aerodynamic applications we had in mind. Challenge II was not considered. The model has not received any major upgrades in its eight years. The rotation/curvature term proposed by [51] has attracted little attention. Current work is directed at DES. Other extensions such as nonlinear and realizable constitutive relations are either not mature enough, or have too little effect to be published.

The S-A model exceeded expectations, and is now useful to a rather large user base. Version control was maintained and frequent upgrades, which are costly for the users, avoided. Its numerical stability is very satisfactory, and in the author's opinion its performance for Challenge I is competitive enough with even the best two- to four-equation models to justify the confidence of users, and the role of "default model" (however, codes should *not* offer only one model!). The model is not sophisticated, but it is not inaccurate.

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